Digitalizing beauty: On NFT art and its price

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Motivation

- NFT art market
  - Market cap US$ 41 billion in 2021: 30 %↑
  - Sales of contemporary art market: 22%↓
  - Some stylized facts of contemporary art trading
    - Infrequent trading
    - Price inequality
    - Illiquidity
    - Centralized patrons
    - Less transparency in pricing
    - Independent marketplaces
- What are these outliers

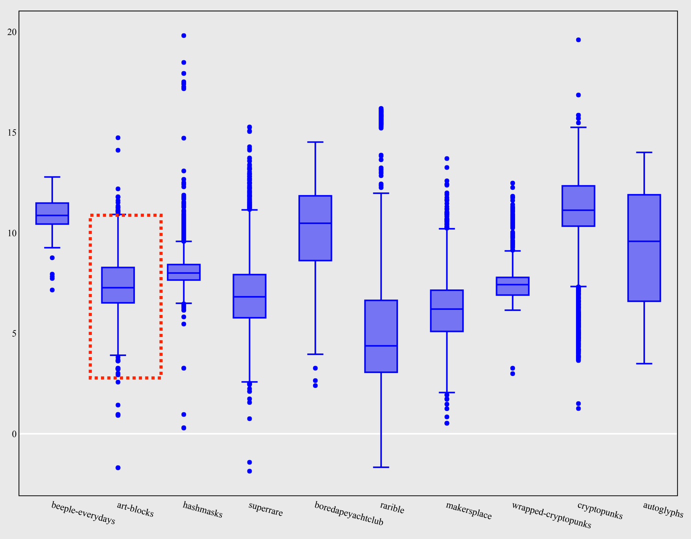


Figure 1. Log price box plot for top 10 collections

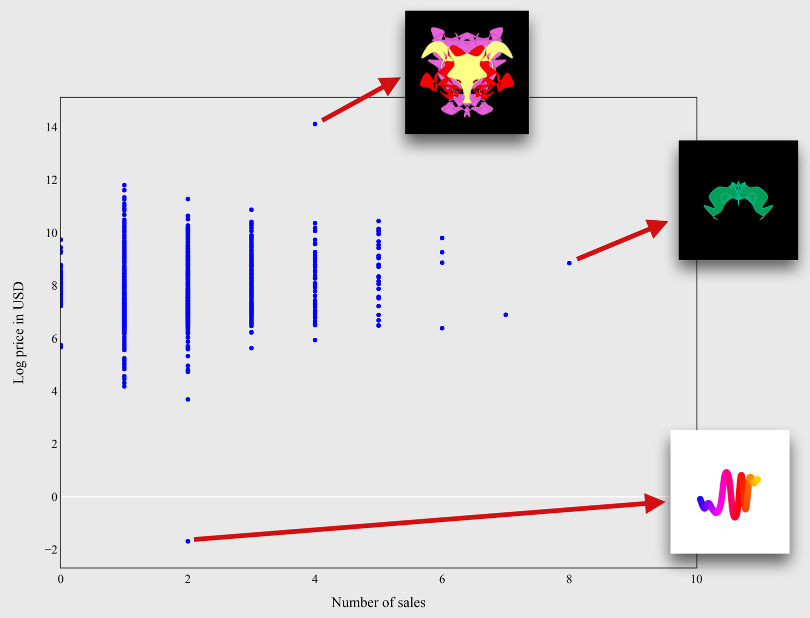


Figure 2. ArtBlocks sales and prices

Statistical Regularities

- Color quantification (K means)

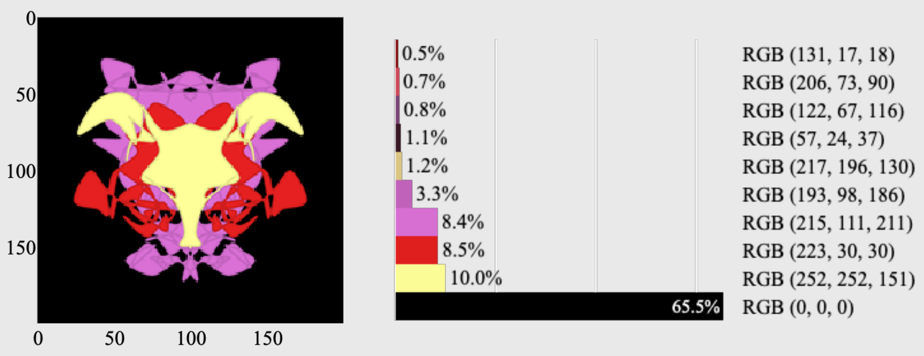


Figure 3. Color palette for ArtBlocks #17225316

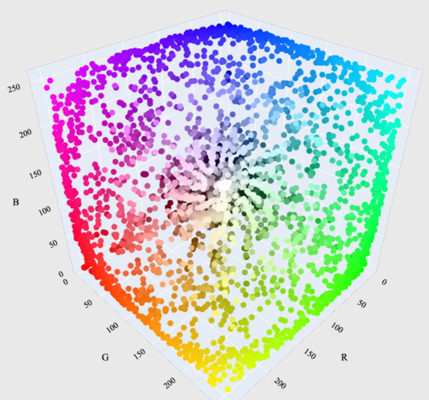
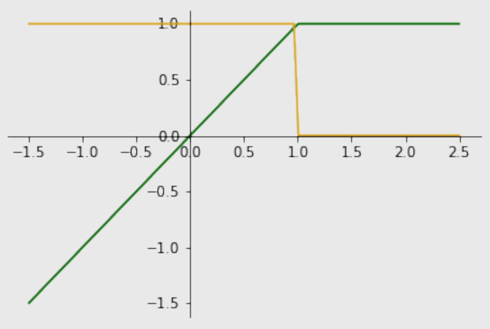
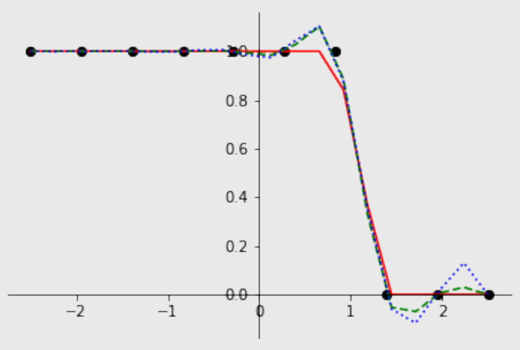


Figure 4. Dominant colors from 3,964 artworks



- Luminance:
  Perceived brightness to RGB colour
  i.e. Hue-saturation-brightness
  $\text{HSB} = \sqrt{0.299R^2 + 0.587G^2 + 0.114B^2}$
- Composition-level statistics



- Pairwise spatial statistics

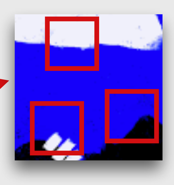
For a set of images $\cup_{x,y \in \mathbb{R}^2} \{u(x,y)\}$

- Asymmetry (Skewness)

$$\text{Skew} = (\#x)^{-1}(\#y)^{-1} \sum_x \sum_y \left[\frac{u(x,y)-\mu}{\sigma}\right]^3$$

- Sparsness (Kurtosis)

$$\text{Kurt} = (\#x)^{-1}(\#y)^{-1} \sum_x \sum_y \left[\frac{u(x,y)-\mu}{\sigma}\right]^4$$



- Energy Spectral Density (ESD)
  Energy of signal $E = \sum_{fr_x} \sum_{fr_y} |\hat{u}_d(fr_x, fr_y)|^2 \Delta fr_x \Delta fr_y$

$$\hat{u}_d(fr_x, fr_y) = \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} u(x,y)\exp\{-i2\pi(fr_x x + fr_y y)\} \text{ (DF)}$$

$|\hat{u}_d(fr_x, fr_y)|^2$ ESD at normalized frequencies fr_x and fr_y

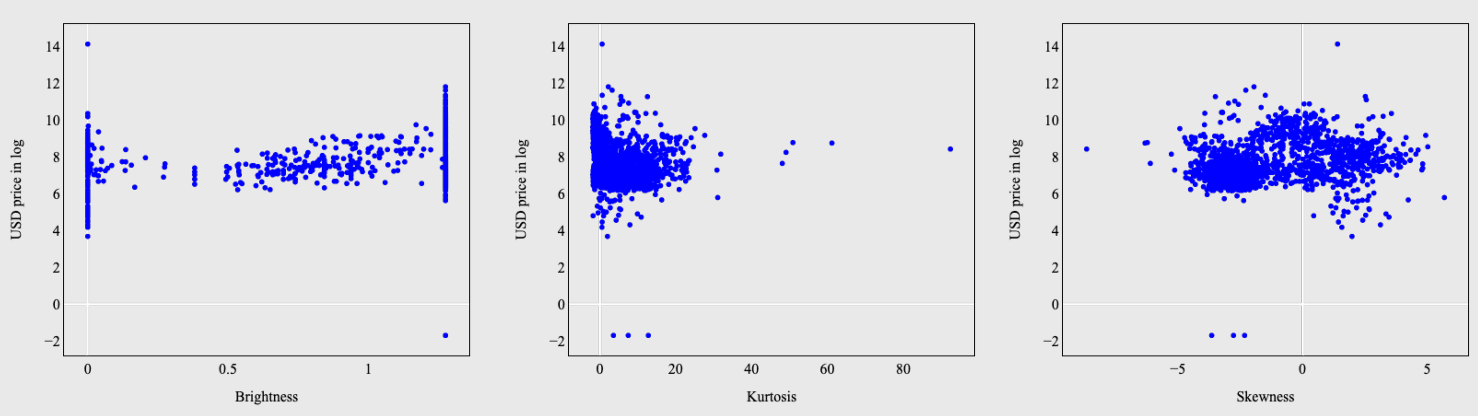


Figure 5. Image statistics vs. Prices

Preliminary Result

- Spatial Energy Spectrum

ESD < > pairwise correlation

$$\left|\sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} u(x,y)\exp\{-i2\pi fr(x+y)\}\right|^2 <> \sum_x \sum_y r_{uu}(x,y)\exp\{-i2\pi fr(x+y)\}$$

assuming same fr and $r_{uu}(x,y)$ autocorrelation of $u(x,y)$

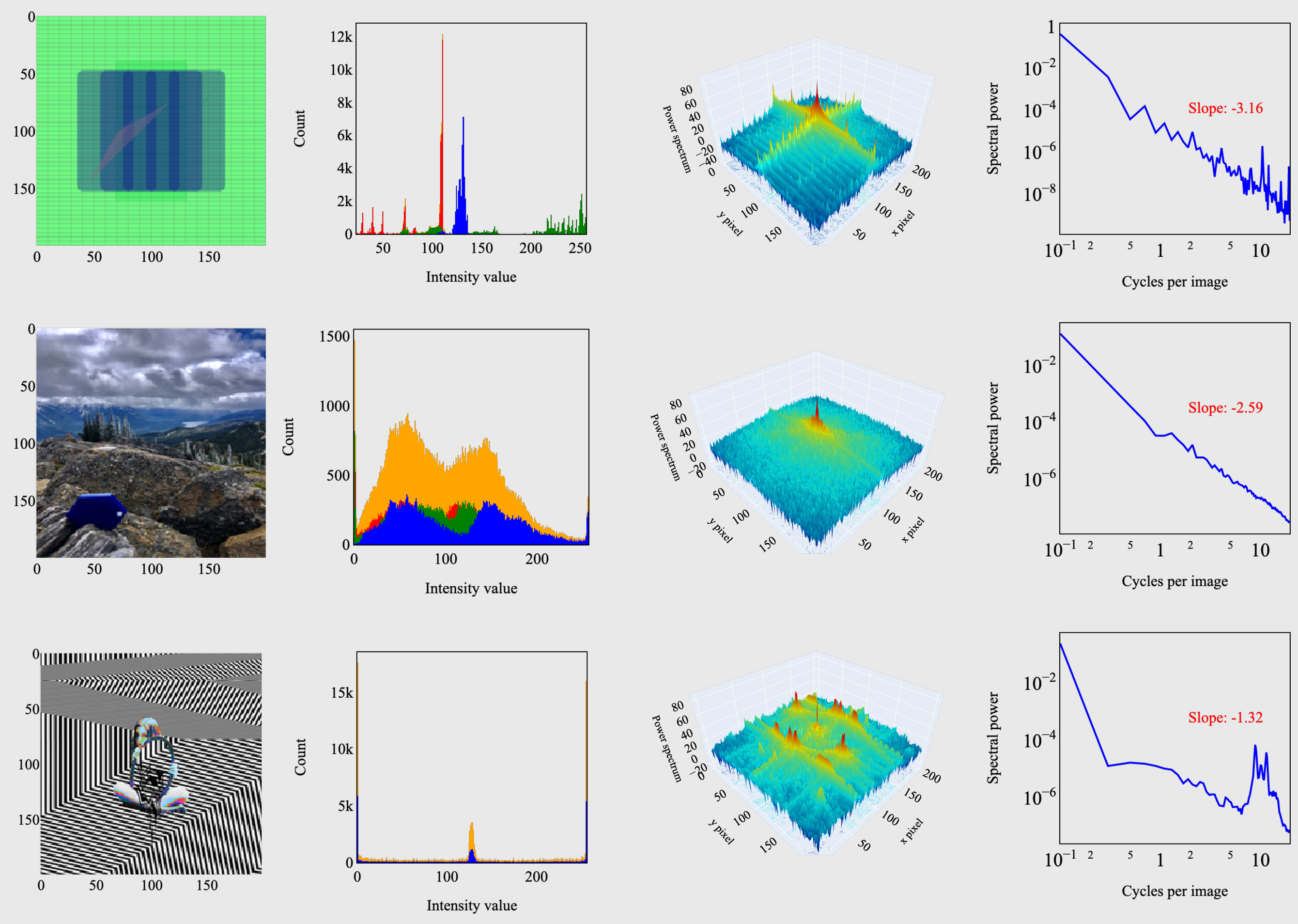
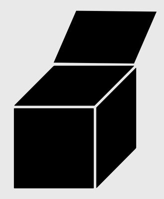


Figure 5. Algo-generated, real object and re-edited images

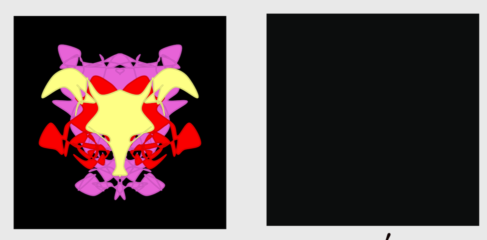
Neural Network



- Interpretability

Learned Features	What features has the NN learned?
Pixel Attribution	How did each pixel contribute to a prediction?
Concepts	Which abstract concepts has the NN learned?
Adversarial Learning	How can we trick the NN?
Influential Instances	How influential is a training point for a certain prediction?

- Integrated gradient
  Sundararajan, Taly & Yan (2017)
  - Input-output pairs $\{(x_i, f(x_i))\}_{i=1}^n \in \mathbb{R}^n \times [0,1]$
  - Network classifier $f: \mathbb{R}^n \to [0,1]$
  - Basis functions $\{\phi_j(x)\}_j^M \in \mathbb{R}^m$



x x'

Attribution $a_f(x, x')$ is contributions of x to $f(x)$ relatively to baseline input x'

Baseline? $\{x_i'\}_{i=1}^n \in \mathbb{R}^n : a_f(x', x') = (0, \ldots, 0)^\top >$ In object recognition:

IG Axioms - I

1. Sensitivity $\forall x_i \neq x_i', \exists \phi_j(x_i) \neq \phi_j(x_i') : f(x_i) \neq f(x') \wedge a_f(x_i, x_i') = a_f(x_i', x_i')$
   ReLU $f(x) = 1 - \max(0, 1-x)$
   Suppose $x' = 0,\ x = 2,\ a_f(x, x') = \dfrac{\delta f}{\delta x}$
2. Implementation Invariance $\forall f, g \in \mathcal{F},\ f(x) = g(x) \wedge a_f(x, x') = a_g(x, x')$

$$IG_i(x) \overset{def}{=} (x_i - x_i') \int_0^1 \frac{\delta f(x' + \alpha(x - x'))}{\delta x_i} d\alpha$$

satisfies 1. and 2. where $\alpha \in [0,1]$ allows the linear interpolation between baseline and original image

IG Axioms - II

Proposition

$f: \mathbb{R}^n \to [0,1]$ is differentiable almost everywhere $> \sum_i IG_i(x) = f(x) - f(x')$

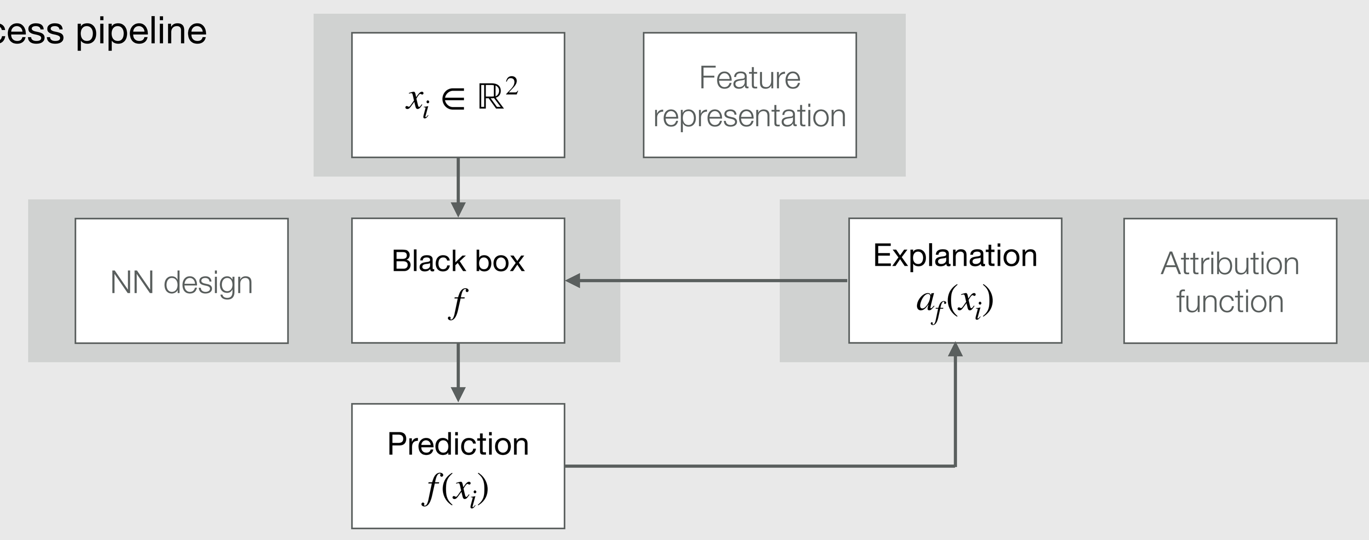
Generalization - Path Methods (PM)

Let $\gamma = (\gamma_1, \ldots, \gamma_n) : [0,1] \to \mathbb{R}^n$ be a smooth function specifying a path in $\mathbb{R}^n$ from x' to x and $\alpha \in [0,1]$, then $PM_i^\gamma(x) \overset{def}{=} \int_0^1 \frac{\delta f(\gamma(\alpha))}{\gamma_i(\alpha)} \frac{\delta \gamma_i(\alpha)}{\delta \alpha} d\alpha$

(ReLU $a_f(x, x')$ cont.) $\gamma(\alpha)$ linear, quadratic, cubic, …

Future Work

Process pipeline



Reference

- Graham, D. J., & Redies, C. (2010). Statistical regularities in art: Relations with visual coding and perception. Vision research, 50(16), 1503-1509.
- Redies, C., Hänisch, J., Blickhan, M., & Denzler, J. (2007). Artists portray human faces with the Fourier statistics of complex natural scenes. Network: Computation in Neural Systems, 18(3), 235-248.
- Schich, M., Huemer, C., Adamczyk, P., Manovich, L., & Liu, Y. Y. (2017). Network dimensions in the getty provenance index. arXiv preprint arXiv:1706.02804.